# Review of Algebra

**Collection Editor:**
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**CONNEXIONS**

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# Chapter 1

# Arithmetic Review: Factors, Products, and Exponents[1]

## 1.1 Overview

- Factors
- Exponential Notation

## 1.2 Factors

Let's begin our review of arithmetic by recalling the meaning of multiplication for whole numbers (the counting numbers and zero).

**Multiplication**

**Multiplication** is a description of repeated addition.

In the addition

$7 + 7 + 7 + 7$

the number 7 is repeated as an **addend*** 4 **times.** Therefore, we say we have **four times seven** and describe it by writing

$4 \cdot 7$

The raised dot between the numbers 4 and 7 indicates multiplication. The dot directs us to multiply the two numbers that it separates. In algebra, the dot is preferred over the symbol × to denote multiplication because the letter $x$ is often used to represent a number. Thus,

$4 \cdot 7 = 7 + 7 + 7 + 7$

**Factors and Products**

In a multiplication, the numbers being multiplied are called **factors.** The result of a multiplication is called the **product.** For example, in the multiplication

$4 \cdot 7 = 28$

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[1] This content is available online at <http://cnx.org/content/m18882/1.5/>.

the numbers 4 and 7 are factors, and the number 28 is the product. We say that 4 and 7 are factors of 28. (They are not the only factors of 28. Can you think of others?)

Now we know that

$$(\text{factor}) \cdot (\text{factor}) = \text{product}$$

This indicates that a first number is a factor of a second number if the first number divides into the second number with no remainder. For example, since

$$4 \cdot 7 = 28$$

both 4 and 7 are factors of 28 since both 4 and 7 divide into 28 with no remainder.

## 1.3 Exponential Notation

Quite often, a particular number will be repeated as a factor in a multiplication. For example, in the multiplication

$$7 \cdot 7 \cdot 7 \cdot 7$$

the number 7 is repeated as a factor 4 times. We describe this by writing $7^4$. Thus,

$$7 \cdot 7 \cdot 7 \cdot 7 = 7^4$$

The repeated factor is the lower number (the base), and the number recording how many times the factor is repeated is the higher number (the superscript). The superscript number is called an **exponent.**

**Exponent**

An **exponent** is a number that records how many times the number to which it is attached occurs as a factor in a multiplication.

## 1.4 Sample Set A

For Examples 1, 2, and 3, express each product using exponents.

**Example 1.1**

$3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$. Since 3 occurs as a factor 6 times,

$$3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 3^6$$

**Example 1.2**

$8 \cdot 8$. Since 8 occurs as a factor 2 times,

$$8 \cdot 8 = 8^2$$

**Example 1.3**

$5 \cdot 5 \cdot 5 \cdot 9 \cdot 9$. Since 5 occurs as a factor 3 times, we have $5^3$. Since 9 occurs as a factor 2 times, we have $9^2$. We should see the following replacements.

$$\underbrace{5 \cdot 5 \cdot 5}_{5^3} \cdot \underbrace{9 \cdot 9}_{9^2}$$

Then we have

$5 \cdot 5 \cdot 5 \cdot 9 \cdot 9 = 5^3 \cdot 9^2$

**Example 1.4**
Expand $3^5$. The base is 3 so it is the repeated factor. The exponent is 5 and it records the number of times the base 3 is repeated. Thus, 3 is to be repeated as a factor 5 times.

$3^5 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$

**Example 1.5**
Expand $6^2 \cdot 10^4$. The notation $6^2 \cdot 10^4$ records the following two facts: 6 is to be repeated as a factor 2 times and 10 is to be repeated as a factor 4 times. Thus,

$6^2 \cdot 10^4 = 6 \cdot 6 \cdot 10 \cdot 10 \cdot 10 \cdot 10$

## 1.5 Exercises

For the following problems, express each product using exponents.

**Exercise 1.1** ***(Solution on p. 5.)***
$8 \cdot 8 \cdot 8$

**Exercise 1.2**
$12 \cdot 12 \cdot 12 \cdot 12 \cdot 12$

**Exercise 1.3** ***(Solution on p. 5.)***
$5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5$

**Exercise 1.4**
$1 \cdot 1$

**Exercise 1.5** ***(Solution on p. 5.)***
$3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 4 \cdot 4$

**Exercise 1.6**
$8 \cdot 8 \cdot 8 \cdot 15 \cdot 15 \cdot 15 \cdot 15$

**Exercise 1.7** ***(Solution on p. 5.)***
$2 \cdot 2 \cdot 2 \cdot 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9$

**Exercise 1.8**
$3 \cdot 3 \cdot 10 \cdot 10 \cdot 10$

**Exercise 1.9** ***(Solution on p. 5.)***
Suppose that the letters $x$ and $y$ are each used to represent numbers. Use exponents to express the following product.

$x \cdot x \cdot x \cdot y \cdot y$

**Exercise 1.10**
Suppose that the letters $x$ and $y$ are each used to represent numbers. Use exponents to express the following product.

$x \cdot x \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y$

For the following problems, expand each product (do not compute the actual value).

**Exercise 1.11** ***(Solution on p. 5.)***
$3^4$

**Exercise 1.12**
$4^3$

**Exercise 1.13** *(Solution on p. 5.)*
$2^5$

**Exercise 1.14**
$9^6$

**Exercise 1.15** *(Solution on p. 5.)*
$5^3 \cdot 6^2$

**Exercise 1.16**
$2^7 \cdot 3^4$

**Exercise 1.17** *(Solution on p. 5.)*
$x^4 \cdot y^4$

**Exercise 1.18**
$x^6 \cdot y^2$

For the following problems, specify all the whole number factors of each number. For example, the complete set of whole number factors of 6 is 1, 2, 3, 6.

**Exercise 1.19** *(Solution on p. 5.)*
20

**Exercise 1.20**
14

**Exercise 1.21** *(Solution on p. 5.)*
12

**Exercise 1.22**
30

**Exercise 1.23** *(Solution on p. 5.)*
21

**Exercise 1.24**
45

**Exercise 1.25** *(Solution on p. 5.)*
11

**Exercise 1.26**
17

**Exercise 1.27** *(Solution on p. 5.)*
19

**Exercise 1.28**
2

# Solutions to Exercises in Chapter 1

**Solution to Exercise 1.1 (p. 3)**
$8^3$
**Solution to Exercise 1.3 (p. 3)**
$5^7$
**Solution to Exercise 1.5 (p. 3)**
$3^5 \cdot 4^2$
**Solution to Exercise 1.7 (p. 3)**
$2^3 \cdot 9^8$
**Solution to Exercise 1.9 (p. 3)**
$x^3 \cdot y^2$
**Solution to Exercise 1.11 (p. 3)**
$3 \cdot 3 \cdot 3 \cdot 3$
**Solution to Exercise 1.13 (p. 4)**
$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$
**Solution to Exercise 1.15 (p. 4)**
$5 \cdot 5 \cdot 5 \cdot 6 \cdot 6$
**Solution to Exercise 1.17 (p. 4)**
$x \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y$
**Solution to Exercise 1.19 (p. 4)**
1, 2, 4, 5, 10, 20
**Solution to Exercise 1.21 (p. 4)**
1, 2, 3, 4, 6, 12
**Solution to Exercise 1.23 (p. 4)**
1, 3, 7, 21
**Solution to Exercise 1.25 (p. 4)**
1, 11
**Solution to Exercise 1.27 (p. 4)**
1, 19

# Chapter 2

# Arithmetic Review: Prime Factorization[1]

## 2.1 Overview

- Prime And Composite Numbers
- The Fundamental Principle Of Arithmetic
- The Prime Factorization Of A Whole Number

## 2.2 Prime And Composite Numbers

Notice that the only factors of 7 are 1 and 7 itself, and that the only factors of 23 are 1 and 23 itself.

**Prime Number**

A whole number greater than 1 whose only whole number factors are itself and 1 is called a **prime number.**

The first seven prime numbers are

2, 3, 5, 7, 11, 13, and 17

The number 1 is not considered to be a prime number, and the number 2 is the first and only even prime number.

Many numbers have factors other than themselves and 1. For example, the factors of 28 are 1, 2, 4, 7, 14, and 28 (since each of these whole numbers and only these whole numbers divide into 28 without a remainder).

**Composite Numbers**

A whole number that is composed of factors other than itself and 1 is called a **composite number.** Composite numbers are not prime numbers.

Some composite numbers are 4, 6, 8, 10, 12, and 15.

## 2.3 The Fundamental Principle Of Arithmetic

Prime numbers are very important in the study of mathematics. We will use them soon in our study of fractions. We will now, however, be introduced to an important mathematical principle.

**The Fundamental Principle of Arithmetic**

Except for the order of the factors, every whole number, other than 1, can be factored in one and only one way as a product of prime numbers.

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[1] This content is available online at <http://cnx.org/content/m21868/1.5/>.

**Prime Factorization**
When a number is factored so that all its factors are prime numbers, the factorization is called the **prime factorization** of the number.

## 2.4 Sample Set A

**Example 2.1**
Find the prime factorization of 10.

$10 = 2 \cdot 5$

Both 2 and 5 are prime numbers. Thus, $2 \cdot 5$ is the prime factorization of 10.

**Example 2.2**
Find the prime factorization of 60.

$$\begin{aligned} 60 &= 2 \cdot 30 && 30 \text{ is not prime. } 30 = 2 \cdot 15 \\ &= 2 \cdot 2 \cdot 15 && 15 \text{ is not prime. } 15 = 3 \cdot 5 \\ &= 2 \cdot 2 \cdot 3 \cdot 5 && \text{We'll use exponents. } 2 \cdot 2 = 2^2 \\ &= 2^2 \cdot 3 \cdot 5 \end{aligned}$$

The numbers 2, 3, and 5 are all primes. Thus, $2^2 \cdot 3 \cdot 5$ is the prime factorization of 60.

**Example 2.3**
Find the prime factorization of 11.

11 is a prime number. Prime factorization applies only to composite numbers.

## 2.5 The Prime Factorization Of A Whole Number

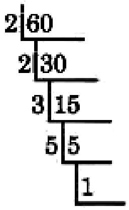
The following method provides a way of finding the prime factorization of a whole number. The examples that follow will use the method and make it more clear.

1. Divide the number repeatedly by the smallest prime number that will divide into the number without a remainder.
2. When the prime number used in step 1 no longer divides into the given number without a remainder, repeat the process with the next largest prime number.
3. Continue this process until the quotient is 1.
4. The prime factorization of the given number is the product of all these prime divisors.

## 2.6 Sample Set B

**Example 2.4**
Find the prime factorization of 60.

Since 60 is an even number, it is divisible by 2. We will repeatedly divide by 2 until we no longer can (when we start getting a remainder). We shall divide in the following way.



30 is divisible by 2 again.

15 is not divisible by 2, but is divisible by 3, the next largest prime.

5 is not divisible by 3, but is divisible by 5, the next largest prime.

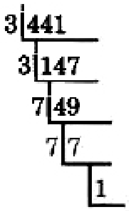
The quotient is 1 so we stop the division process.

The prime factorization of 60 is the product of all these divisors.

$$\begin{aligned} 60 &= 2 \cdot 2 \cdot 3 \cdot 5 \qquad \text{We will use exponents when possible.} \\ 60 &= 2^2 \cdot 3 \cdot 5 \end{aligned}$$

**Example 2.5**
Find the prime factorization of 441.

Since 441 is an odd number, it is not divisible by 2. We'll try 3, the next largest prime.



147 is divisible by 3.

49 is not divisible by 3 nor by 5, but by 7.

7 is divisible by 7.

The quotient is 1 so we stop the division process.

The prime factorization of 441 is the product of all the divisors.

$$\begin{aligned} 441 &= 3 \cdot 3 \cdot 7 \cdot 7 \qquad \text{We will use exponents when possible.} \\ 441 &= 3^2 \cdot 7^2 \end{aligned}$$

## 2.7 Exercises

For the following problems, determine which whole numbers are prime and which are composite.

**Exercise 2.1** *(Solution on p. 11.)*
23

**Exercise 2.2**
25

**Exercise 2.3** *(Solution on p. 11.)*
27

**Exercise 2.4**
2

**Exercise 2.5** *(Solution on p. 11.)*
3

**Exercise 2.6**
5

**Exercise 2.7** ***(Solution on p. 11.)***
7

**Exercise 2.8**
9

**Exercise 2.9** ***(Solution on p. 11.)***
11

**Exercise 2.10**
34

**Exercise 2.11** ***(Solution on p. 11.)***
55

**Exercise 2.12**
63

**Exercise 2.13** ***(Solution on p. 11.)***
1044

**Exercise 2.14**
339

**Exercise 2.15** ***(Solution on p. 11.)***
209

For the following problems, find the prime factorization of each whole number. Use exponents on repeated factors.

**Exercise 2.16**
26

**Exercise 2.17** ***(Solution on p. 11.)***
38

**Exercise 2.18**
54

**Exercise 2.19** ***(Solution on p. 11.)***
62

**Exercise 2.20**
56

**Exercise 2.21** ***(Solution on p. 11.)***
176

**Exercise 2.22**
480

**Exercise 2.23** ***(Solution on p. 11.)***
819

**Exercise 2.24**
2025

**Exercise 2.25** ***(Solution on p. 11.)***
148,225

# Solutions to Exercises in Chapter 2

**Solution to Exercise 2.1 (p. 9)**
prime

**Solution to Exercise 2.3 (p. 9)**
composite

**Solution to Exercise 2.5 (p. 9)**
prime

**Solution to Exercise 2.7 (p. 10)**
prime

**Solution to Exercise 2.9 (p. 10)**
prime

**Solution to Exercise 2.11 (p. 10)**
composite

**Solution to Exercise 2.13 (p. 10)**
composite

**Solution to Exercise 2.15 (p. 10)**
composite

**Solution to Exercise 2.17 (p. 10)**
$2 \cdot 19$

**Solution to Exercise 2.19 (p. 10)**
$2 \cdot 31$

**Solution to Exercise 2.21 (p. 10)**
$2^4 \cdot 11$

**Solution to Exercise 2.23 (p. 10)**
$3^2 \cdot 7 \cdot 13$

**Solution to Exercise 2.25 (p. 10)**
$5^2 \cdot 7^2 \cdot 11^2$

# Chapter 3

# Arithmetic Review: The Least Common Multiple[1]

## 3.1 Overview

- Multiples
- Common Multiples
- The Least Common Multiple (LCM)
- Finding The Least Common Multiple

## 3.2 Multiples

**Multiples**
When a whole number is multiplied by other whole numbers, with the exception of Multiples zero, the resulting products are called **multiples** of the given whole number.

| **Multiples of 2** | | **Multiples of 3** | | **Multiples of 8** | | **Multiples of 10** |
|---|---|---|---|---|---|---|
| $2 \cdot 1=2$ | | $3 \cdot 1=3$ | | $8 \cdot 1=8$ | | $10 \cdot 1=10$ |
| $2 \cdot 2=4$ | | $3 \cdot 2=6$ | | $8 \cdot 2=16$ | | $10 \cdot 2=20$ |
| $2 \cdot 3=6$ | | $3 \cdot 3=9$ | | $8 \cdot 3=24$ | | $10 \cdot 3=30$ |
| $2 \cdot 4=8$ | | $3 \cdot 4=12$ | | $8 \cdot 4=32$ | | $10 \cdot 4=40$ |
| $2 \cdot 5=10$ | | $3 \cdot 5=15$ | | $8 \cdot 5=40$ | | $10 \cdot 5=50$ |
| ... | | ... | | ... | | ... |

**Table 3.1**

## 3.3 Common Multiples

There will be times when we are given two or more whole numbers and we will need to know if there are any multiples that are common to each of them. If there are, we will need to know what they are. For example, some of the multiples that are common to 2 and 3 are 6, 12, and 18.

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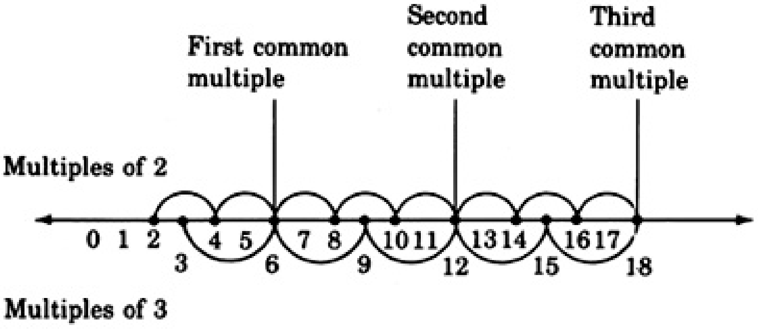
[1] This content is available online at <http://cnx.org/content/m21870/1.6/>.

## 3.4 Sample Set A

**Example 3.1**
We can visualize common multiples using the number line.



Notice that the common multiples can be divided by both whole numbers.

## 3.5 The Least Common Multiple (LCM)

Notice that in our number line visualization of common multiples (above) the first common multiple is also the smallest, or **least common multiple,** abbreviated by **LCM.**

**Least Common Multiple**
The **least common multiple, LCM,** of two or more whole numbers is the smallest whole number that each of the given numbers will divide into without a remainder.

## 3.6 Finding The Least Common Multiple

**Finding the LCM**
To find the LCM of two or more numbers,

1. Write the prime factorization of each number, using exponents on repeated factors.
2. Write each base that appears in each of the prime factorizations.
3. To each base, attach the largest exponent that appears on it in the prime factorizations.
4. The LCM is the product of the numbers found in step 3.

## 3.7 Sample Set B

Find the LCM of the following number.

**Example 3.2**
9 and 12

1. $$\begin{array}{rcl} 9 & = & 3 \cdot 3 = 3^2 \\ 12 & = & 2 \cdot 6 = 2 \cdot 2 \cdot 3 = 2^2 \cdot 3 \end{array}$$
2. The bases that appear in the prime factorizations are 2 and 3.
3. The largest exponents appearing on 2 and 3 in the prime factorizations are, respectively, 2 and 2 (or $2^2$ from 12, and $3^2$ from 9).

4. The LCM is the product of these numbers.

$$\text{LCM} = 2^2 \cdot 3^2 = 4 \cdot 9 = 36$$

Thus, 36 is the smallest number that both 9 and 12 divide into without remainders.

**Example 3.3**
90 and 630

1.
$$\begin{aligned} 90 &= 2 \cdot 45 &&= 2 \cdot 3 \cdot 15 &&= 2 \cdot 3 \cdot 3 \cdot 5 &&= 2 \cdot 3^2 \cdot 5 \\ 630 &= 2 \cdot 315 &&= 2 \cdot 3 \cdot 105 &&= 2 \cdot 3 \cdot 3 \cdot 35 &&= 2 \cdot 3 \cdot 3 \cdot 5 \cdot 7 \\ & && && &&= 2 \cdot 3^2 \cdot 5 \cdot 7 \end{aligned}$$

2. The bases that appear in the prime factorizations are 2, 3, 5, and 7.
3. The largest exponents that appear on 2, 3, 5, and 7 are, respectively, 1, 2, 1, and 1.

   $2^1$ from either 90 or 630
   $3^2$ from either 90 or 630
   $5^1$ from either 90 or 630
   $7^1$ from 630

4. The LCM is the product of these numbers.

$$\text{LCM} = 2 \cdot 3^2 \cdot 5 \cdot 7 = 2 \cdot 9 \cdot 5 \cdot 7 = 630$$

Thus, 630 is the smallest number that both 90 and 630 divide into with no remainders.

**Example 3.4**
33, 110, and 484

1.
$$\begin{aligned} 33 &= 3 \cdot 11 \\ 110 &= 2 \cdot 55 &&= 2 \cdot 5 \cdot 11 \\ 484 &= 2 \cdot 242 &&= 2 \cdot 2 \cdot 121 = 2 \cdot 2 \cdot 11 \cdot 11 = 2^2 \cdot 11^2 \end{aligned}$$

2. The bases that appear in the prime factorizations are 2, 3, 5, and 11.
3. The largest exponents that appear on 2, 3, 5, and 11 are, respectively, 2, 1, 1, and 2.

   $2^2$ from 484
   $3^1$ from 33
   $5^1$ from 110
   $11^2$ from 484

4. The LCM is the product of these numbers.

$$\begin{aligned} \text{LCM} &= 2^2 \cdot 3 \cdot 5 \cdot 11^2 \\ &= 4 \cdot 3 \cdot 5 \cdot 121 \\ &= 7260 \end{aligned}$$

Thus, 7260 is the smallest number that 33, 110, and 484 divide into without remainders.

## 3.8 Exercises

For the following problems, find the least common multiple of given numbers.

**Exercise 3.1** *(Solution on p. 18.)*
8, 12

**Exercise 3.2**
8, 10

**Exercise 3.3** *(Solution on p. 18.)*
6, 12

**Exercise 3.4**
9, 18

**Exercise 3.5** *(Solution on p. 18.)*
5, 6

**Exercise 3.6**
7, 9

**Exercise 3.7** *(Solution on p. 18.)*
28, 36

**Exercise 3.8**
24, 36

**Exercise 3.9** *(Solution on p. 18.)*
28, 42

**Exercise 3.10**
20, 24

**Exercise 3.11** *(Solution on p. 18.)*
25, 30

**Exercise 3.12**
24, 54

**Exercise 3.13** *(Solution on p. 18.)*
16, 24

**Exercise 3.14**
36, 48

**Exercise 3.15** *(Solution on p. 18.)*
15, 21

**Exercise 3.16**
7, 11, 33

**Exercise 3.17** *(Solution on p. 18.)*
8, 10, 15

**Exercise 3.18**
4, 5, 21

**Exercise 3.19** *(Solution on p. 18.)*
45, 63, 98

**Exercise 3.20**
15, 25, 40

**Exercise 3.21** *(Solution on p. 18.)*
12, 16, 20

**Exercise 3.22**
12, 16, 24

**Exercise 3.23** *(Solution on p. 18.)*
12, 16, 24, 36

**Exercise 3.24**
6, 9, 12, 18

**Exercise 3.25** *(Solution on p. 18.)*
8, 14, 28, 32

## Solutions to Exercises in Chapter 3

**Solution to Exercise 3.1 (p. 16)**
$2^3 \cdot 3$

**Solution to Exercise 3.3 (p. 16)**
$2^2 \cdot 3$

**Solution to Exercise 3.5 (p. 16)**
$2 \cdot 3 \cdot 5$

**Solution to Exercise 3.7 (p. 16)**
$2^2 \cdot 3^2 \cdot 7$

**Solution to Exercise 3.9 (p. 16)**
$2^2 \cdot 3 \cdot 7$

**Solution to Exercise 3.11 (p. 16)**
$2 \cdot 3 \cdot 5^2$

**Solution to Exercise 3.13 (p. 16)**
$2^4 \cdot 3$

**Solution to Exercise 3.15 (p. 16)**
$3 \cdot 5 \cdot 7$

**Solution to Exercise 3.17 (p. 16)**
$2^3 \cdot 3 \cdot 5$

**Solution to Exercise 3.19 (p. 16)**
$2 \cdot 3^2 \cdot 5 \cdot 7^2$

**Solution to Exercise 3.21 (p. 16)**
$2^4 \cdot 3 \cdot 5$

**Solution to Exercise 3.23 (p. 17)**
$2^4 \cdot 3^2$

**Solution to Exercise 3.25 (p. 17)**
$2^5 \cdot 7$

# Chapter 4

# Arithmetic Review: Equivalent Fractions[1]

## 4.1 Overview

- Equivalent Fractions
- Reducing Fractions To Lowest Terms
- Raising Fractions To Higher Terms

## 4.2 Equivalent Fractions

**Equivalent Fractions**
Fractions that have the same value are called **equivalent fractions.**

For example, $\frac{2}{3}$ and $\frac{4}{6}$ represent the same part of a whole quantity and are therefore equivalent. Several more collections of equivalent fractions are listed below.

**Example 4.1**
$\frac{15}{25}, \frac{12}{20}, \frac{3}{5}$

**Example 4.2**
$\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}$

**Example 4.3**
$\frac{7}{6}, \frac{14}{12}, \frac{21}{18}, \frac{28}{24}, \frac{35}{30}$

## 4.3 Reducing Fractions To Lowest Terms

**Reduced to Lowest Terms**
It is often useful to convert one fraction to an equivalent fraction that has reduced values in the numerator and denominator. When a fraction is converted to an equivalent fraction that has the smallest numerator and denominator in the collection of equivalent fractions, it is said to be **reduced to lowest terms.** The conversion process is called **reducing a fraction.**

We can reduce a fraction to lowest terms by

1. Expressing the numerator and denominator as a product of prime numbers. (Find the prime factorization of the numerator and denominator. See Section (Chapter 2) for this technique.)

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[1] This content is available online at <http://cnx.org/content/m21861/1.4/>.

2. Divide the numerator and denominator by all common factors. (This technique is commonly called "cancelling.")

## 4.4 Sample Set A

Reduce each fraction to lowest terms.

**Example 4.4**

$$\begin{aligned} \frac{6}{18} &= \frac{2\cdot 3}{2\cdot 3\cdot 3} \\ &= \frac{\cancel{2}\cdot\cancel{3}}{\cancel{2}\cdot\cancel{3}\cdot 3} && \text{2 and 3 are common factors.} \\ &= \frac{1}{3} \end{aligned}$$

**Example 4.5**

$$\begin{aligned} \frac{16}{20} &= \frac{2\cdot 2\cdot 2\cdot 2}{2\cdot 2\cdot 5} \\ &= \frac{\cancel{2}\cdot\cancel{2}\cdot 2\cdot 2}{\cancel{2}\cdot\cancel{2}\cdot 5} && \text{2 is the only common factor.} \\ &= \frac{4}{5} \end{aligned}$$

**Example 4.6**

$$\begin{aligned} \frac{56}{70} &= \frac{2\cdot 4\cdot 7}{2\cdot 5\cdot 7} \\ &= \frac{\cancel{2}\cdot 4\cdot\cancel{7}}{\cancel{2}\cdot 5\cdot\cancel{7}} && \text{2 and 7 are common factors.} \\ &= \frac{4}{5} \end{aligned}$$

**Example 4.7**

$$\frac{8}{15} = \frac{2\cdot 2\cdot 2}{3\cdot 5} \qquad \text{There are no common factors.}$$

Thus, $\frac{8}{15}$ is reduced to lowest terms.

## 4.5 Raising a Fraction to Higher Terms

Equally important as reducing fractions is **raising fractions to higher terms.** Raising a fraction to higher terms is the process of constructing an equivalent fraction that has higher values in the numerator and denominator. The higher, equivalent fraction is constructed by multiplying the original fraction by 1.

Notice that $\frac{3}{5}$ and $\frac{9}{15}$ are equivalent, that is $\frac{3}{5} = \frac{9}{15}$. Also,

$$\frac{3}{5}\cdot 1 = \frac{3}{5}\cdot\frac{3}{3} = \frac{3\cdot 3}{5\cdot 3} = \frac{9}{15}$$

$$1 = \frac{3}{3}$$

This observation helps us suggest the following method for raising a fraction to higher terms.

**Raising a Fraction to Higher Terms**

A fraction can be raised to higher terms by multiplying both the numerator and denominator by the same nonzero number.

For example, $\frac{3}{4}$ can be raised to $\frac{24}{32}$ by multiplying both the numerator and denominator by 8, that is, multiplying by 1 in the form $\frac{8}{8}$.

$$\frac{3}{4} = \frac{3 \cdot 8}{4 \cdot 8} = \frac{24}{32}$$

How did we know to choose 8 as the proper factor? Since we wish to convert 4 to 32 by multiplying it by some number, we know that 4 must be a factor of 32. This means that 4 divides into 32. In fact, $32 \div 4 = 8$. We divided the original denominator into the new, specified denominator to obtain the proper factor for the multiplication.

## 4.6 Sample Set B

Determine the missing numerator or denominator.

**Example 4.8**

$\frac{3}{7} = \frac{?}{35}$. Divide the original denominator, 7, into the new denominator, $35. 35 \div 7 = 5$.

Multiply the original numerator by 5.

$\frac{3}{7} = \frac{3 \cdot 5}{7 \cdot 5} = \frac{15}{35}$

**Example 4.9**

$\frac{5}{6} = \frac{45}{?}$. Divide the original numerator, 5, into the new numerator, $45. 45 \div 5 = 9$.

Multiply the original denominator by 9.

$\frac{5}{6} = \frac{5 \cdot 9}{6 \cdot 9} = \frac{45}{54}$

## 4.7 Exercises

For the following problems, reduce, if possible, each fraction lowest terms.

**Exercise 4.1** ***(Solution on p. 24.)***

$\frac{6}{8}$

**Exercise 4.2**

$\frac{5}{10}$

**Exercise 4.3** ***(Solution on p. 24.)***

$\frac{6}{14}$

**Exercise 4.4**

$\frac{4}{14}$

**Exercise 4.5** ***(Solution on p. 24.)***

$\frac{18}{12}$

**Exercise 4.6**

$\frac{20}{8}$

**Exercise 4.7** ***(Solution on p. 24.)***

$\frac{10}{6}$

**Exercise 4.8**

$\frac{14}{4}$

**Exercise 4.9** *(Solution on p. 24.)*

$\frac{10}{12}$

**Exercise 4.10**

$\frac{32}{28}$

**Exercise 4.11** *(Solution on p. 24.)*

$\frac{36}{10}$

**Exercise 4.12**

$\frac{26}{60}$

**Exercise 4.13** *(Solution on p. 24.)*

$\frac{12}{18}$

**Exercise 4.14**

$\frac{18}{27}$

**Exercise 4.15** *(Solution on p. 24.)*

$\frac{18}{24}$

**Exercise 4.16**

$\frac{32}{40}$

**Exercise 4.17** *(Solution on p. 24.)*

$\frac{11}{22}$

**Exercise 4.18**

$\frac{17}{51}$

**Exercise 4.19** *(Solution on p. 24.)*

$\frac{27}{81}$

**Exercise 4.20**

$\frac{16}{42}$

**Exercise 4.21** *(Solution on p. 24.)*

$\frac{39}{13}$

**Exercise 4.22**

$\frac{44}{11}$

**Exercise 4.23** *(Solution on p. 24.)*

$\frac{121}{132}$

**Exercise 4.24**

$\frac{30}{105}$

**Exercise 4.25** *(Solution on p. 24.)*

$\frac{108}{76}$

For the following problems, determine the missing numerator or denominator.

**Exercise 4.26**

$\frac{1}{3} = \frac{?}{12}$

**Exercise 4.27** *(Solution on p. 24.)*

$\frac{1}{5} = \frac{?}{30}$

**Exercise 4.28**

$\frac{3}{3} = \frac{?}{9}$

**Exercise 4.29** *(Solution on p. 24.)*

$\frac{3}{4} = \frac{?}{16}$

**Exercise 4.30**

$\frac{5}{6} = \frac{?}{18}$

**Exercise 4.31** *(Solution on p. 24.)*

$\frac{4}{5} = \frac{?}{25}$

**Exercise 4.32**

$\frac{1}{2} = \frac{4}{?}$

**Exercise 4.33** *(Solution on p. 24.)*

$\frac{9}{25} = \frac{27}{?}$

**Exercise 4.34**

$\frac{3}{2} = \frac{18}{?}$

**Exercise 4.35** *(Solution on p. 24.)*

$\frac{5}{3} = \frac{80}{?}$

## Solutions to Exercises in Chapter 4

**Solution to Exercise 4.1 (p. 21)**
$\frac{3}{4}$

**Solution to Exercise 4.3 (p. 21)**
$\frac{3}{7}$

**Solution to Exercise 4.5 (p. 21)**
$\frac{3}{2}$

**Solution to Exercise 4.7 (p. 21)**
$\frac{5}{3}$

**Solution to Exercise 4.9 (p. 22)**
$\frac{5}{6}$

**Solution to Exercise 4.11 (p. 22)**
$\frac{18}{5}$

**Solution to Exercise 4.13 (p. 22)**
$\frac{2}{3}$

**Solution to Exercise 4.15 (p. 22)**
$\frac{3}{4}$

**Solution to Exercise 4.17 (p. 22)**
$\frac{1}{2}$

**Solution to Exercise 4.19 (p. 22)**
$\frac{1}{3}$

**Solution to Exercise 4.21 (p. 22)**
3

**Solution to Exercise 4.23 (p. 22)**
$\frac{11}{12}$

**Solution to Exercise 4.25 (p. 22)**
$\frac{27}{19}$

**Solution to Exercise 4.27 (p. 22)**
6

**Solution to Exercise 4.29 (p. 22)**
12

**Solution to Exercise 4.31 (p. 23)**
20

**Solution to Exercise 4.33 (p. 23)**
75

**Solution to Exercise 4.35 (p. 23)**
48

# Chapter 5

# Arithmetic Review: Operations with Fractions[1]

## 5.1 Overview

- Multiplication of Fractions
- Division of Fractions
- Addition and Subtraction of Fractions

## 5.2 Multiplication of Fractions

**Multiplication of Fractions**
To multiply two fractions, multiply the numerators together and multiply the denominators together. Reduce to lowest terms if possible.

**Example 5.1**
For example, multiply $\frac{3}{4} \cdot \frac{1}{6}$.

$$\begin{aligned}
\frac{3}{4} \cdot \frac{1}{6} &= \frac{3 \cdot 1}{4 \cdot 6} \\
&= \frac{3}{24} && \text{Now reduce.} \\
&= \frac{3 \cdot 1}{2 \cdot 2 \cdot 2 \cdot 3} \\
&= \frac{\cancel{3} \cdot 1}{2 \cdot 2 \cdot 2 \cdot \cancel{3}} && \text{3 is the only common factor.} \\
&= \frac{1}{8}
\end{aligned}$$

Notice that we since had to reduce, we nearly started over again with the original two fractions. If we factor first, then cancel, then multiply, we will save time and energy and still obtain the correct product.

## 5.3 Sample Set A

Perform the following multiplications.

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[1] This content is available online at <http://cnx.org/content/m21867/1.4/>.

**Example 5.2**

$$\begin{aligned} \frac{1}{4}\cdot\frac{8}{9} &= \frac{1}{2\cdot 2}\cdot\frac{2\cdot 2\cdot 2}{3\cdot 3} \\ &= \frac{1}{\cancel{2}\cdot\cancel{2}}\cdot\frac{\cancel{2}\cdot\cancel{2}\cdot 2}{3\cdot 3} && \text{2 is a common factor.} \\ &= \frac{1}{1}\cdot\frac{2}{3\cdot 3} \\ &= \frac{1\cdot 2}{1\cdot 3\cdot 3} \\ &= \frac{2}{9} \end{aligned}$$

**Example 5.3**

$$\begin{aligned} \frac{3}{4}\cdot\frac{8}{9}\cdot\frac{5}{12} &= \frac{3}{2\cdot 2}\cdot\frac{2\cdot 2\cdot 2}{3\cdot 3}\cdot\frac{5}{2\cdot 2\cdot 3} \\ &= \frac{\cancel{3}}{\cancel{2}\cdot\cancel{2}}\cdot\frac{\cancel{2}\cdot\cancel{2}\cdot\cancel{2}}{\cancel{3}\cdot 3}\cdot\frac{5}{\cancel{2}\cdot 2\cdot 3} && \text{2 and 3 are common factors.} \\ &= \frac{1\cdot 1\cdot 5}{3\cdot 2\cdot 3} \\ &= \frac{5}{18} \end{aligned}$$

## 5.4 Division of Fractions

**Reciprocals**

Two numbers whose product is 1 are **reciprocals** of each other. For example, since $\frac{4}{5}\cdot\frac{5}{4}=1$, $\frac{4}{5}$ and $\frac{5}{4}$ are reciprocals of each other. Some other pairs of reciprocals are listed below.

$$\frac{2}{7},\frac{7}{2} \qquad \frac{3}{4},\frac{4}{3} \qquad \frac{6}{1},\frac{1}{6}$$

Reciprocals are used in division of fractions.

**Division of Fractions**

To divide a first fraction by a second fraction, multiply the first fraction by the reciprocal of the second fraction. Reduce if possible.

This method is sometimes called the “invert and multiply” method.

## 5.5 Sample Set B

Perform the following divisions.

**Example 5.4**

$$\begin{aligned} &\frac{1}{3}\div\frac{3}{4}. && \text{The divisor is } \tfrac{3}{4}\text{. Its reciprocal is } \tfrac{4}{3}. \\ \frac{1}{3}\div\frac{3}{4} &= \frac{1}{3}\cdot\frac{4}{3} \\ &= \frac{1\cdot 4}{3\cdot 3} \\ &= \frac{4}{9} \end{aligned}$$

**Example 5.5**

$\frac{3}{8} \div \frac{5}{4}$. The divisor is $\frac{5}{4}$. Its reciprocal is $\frac{4}{5}$.

$$\begin{aligned}
\frac{3}{8} \div \frac{5}{4} &= \frac{3}{8} \cdot \frac{4}{5} \\
&= \frac{3}{2 \cdot 2 \cdot 2} \cdot \frac{2 \cdot 2}{5} \\
&= \frac{3}{\cancel{2} \cdot \cancel{2} \cdot 2} \cdot \frac{\cancel{2} \cdot \cancel{2}}{5} && \text{2 is a common factor.} \\
&= \frac{3 \cdot 1}{2 \cdot 5} \\
&= \frac{3}{10}
\end{aligned}$$

**Example 5.6**

$\frac{5}{6} \div \frac{5}{12}$. The divisor is $\frac{5}{12}$. Its reciprocal is $\frac{12}{5}$.

$$\begin{aligned}
\frac{5}{6} \div \frac{5}{12} &= \frac{5}{6} \cdot \frac{12}{5} \\
&= \frac{5}{2 \cdot 3} \cdot \frac{2 \cdot 2 \cdot 3}{5} \\
&= \frac{\cancel{5}}{\cancel{2} \cdot \cancel{3}} \cdot \frac{\cancel{2} \cdot 2 \cdot \cancel{3}}{\cancel{5}} \\
&= \frac{1 \cdot 2}{1} \\
&= 2
\end{aligned}$$

## 5.6 Addition and Subtraction of Fractions

**Fractions with Like Denominators**

To add (or subtract) two or more fractions that have the same denominators, add (or subtract) the numerators and place the resulting sum over the common denominator. Reduce if possible.

CAUTION

Add or subtract only the numerators. Do **not** add or subtract the denominators!

## 5.7 Sample Set C

Find the following sums.

**Example 5.7**

$\frac{3}{7} + \frac{2}{7}$. The denominators are the same. Add the numerators and place the sum over 7.

$\frac{3}{7} + \frac{2}{7} = \frac{3+2}{7} = \frac{5}{7}$

**Example 5.8**

$\frac{7}{9} - \frac{4}{9}$. The denominators are the same. Subtract 4 from 7 and place the difference over 9.

$\frac{7}{9} - \frac{4}{9} = \frac{7-4}{9} = \frac{3}{9} = \frac{1}{3}$

## 5.8

Fractions can only be added or subtracted conveniently if they have like denominators.

**Fractions with Unlike Denominators**

To add or subtract fractions having unlike denominators, convert each fraction to an equivalent fraction having as the denominator the least common multiple of the original denominators.

The least common multiple of the original denominators is commonly referred to as the **least common denominator** (LCD). See Section (Chapter 3) for the technique of finding the least common multiple of several numbers.

## 5.9 Sample Set D

Find each sum or difference.

**Example 5.9**

$\frac{1}{6}+\frac{3}{4}$. The denominators are not alike. Find the LCD of 6 and 4.

$\left\{ \begin{array}{l} 6 = 2 \cdot 3 \\ 4 = 2^2 \end{array} \right.$ The LCD is $2^2 \cdot 3 = 4 \cdot 3 = 12$.

Convert each of the original fractions to equivalent fractions having the common denominator 12.

$\frac{1}{6} = \frac{1 \cdot 2}{6 \cdot 2} = \frac{2}{12} \qquad \frac{3}{4} = \frac{3 \cdot 3}{4 \cdot 3} = \frac{9}{12}$

Now we can proceed with the addition.

$$\begin{aligned} \frac{1}{6}+\frac{3}{4} &= \frac{2}{12}+\frac{9}{12} \\ &= \frac{2+9}{12} \\ &= \frac{11}{12} \end{aligned}$$

**Example 5.10**

$\frac{5}{9}-\frac{5}{12}$. The denominators are not alike. Find the LCD of 9 and 12.

$\left\{ \begin{array}{l} 9 = 3^2 \\ 12 = 2^2 \cdot 3 \end{array} \right.$ The LCD is $2^2 \cdot 3^2 = 4 \cdot 9 = 36$.

Convert each of the original fractions to equivalent fractions having the common denominator 36.

$\frac{5}{9} = \frac{5 \cdot 4}{9 \cdot 4} = \frac{20}{36} \qquad \frac{5}{12} = \frac{5 \cdot 3}{12 \cdot 3} = \frac{15}{36}$

Now we can proceed with the subtraction.

$$\begin{aligned} \frac{5}{9}-\frac{5}{12} &= \frac{20}{36}-\frac{15}{36} \\ &= \frac{20-15}{36} \\ &= \frac{5}{36} \end{aligned}$$

## 5.10 Exercises

For the following problems, perform each indicated operation.

**Exercise 5.1** *(Solution on p. 31.)*

$\frac{1}{3} \cdot \frac{4}{3}$

**Exercise 5.2**
$\frac{1}{3} \cdot \frac{2}{3}$

**Exercise 5.3** ***(Solution on p. 31.)***
$\frac{2}{5} \cdot \frac{5}{6}$

**Exercise 5.4**
$\frac{5}{6} \cdot \frac{14}{15}$

**Exercise 5.5** ***(Solution on p. 31.)***
$\frac{9}{16} \cdot \frac{20}{27}$

**Exercise 5.6**
$\frac{35}{36} \cdot \frac{48}{55}$

**Exercise 5.7** ***(Solution on p. 31.)***
$\frac{21}{25} \cdot \frac{15}{14}$

**Exercise 5.8**
$\frac{76}{99} \cdot \frac{66}{38}$

**Exercise 5.9** ***(Solution on p. 31.)***
$\frac{3}{7} \cdot \frac{14}{18} \cdot \frac{6}{2}$

**Exercise 5.10**
$\frac{14}{15} \cdot \frac{21}{28} \cdot \frac{45}{7}$

**Exercise 5.11** ***(Solution on p. 31.)***
$\frac{5}{9} \div \frac{5}{6}$

**Exercise 5.12**
$\frac{9}{16} \div \frac{15}{8}$

**Exercise 5.13** ***(Solution on p. 31.)***
$\frac{4}{9} \div \frac{6}{15}$

**Exercise 5.14**
$\frac{25}{49} \div \frac{4}{9}$

**Exercise 5.15** ***(Solution on p. 31.)***
$\frac{15}{4} \div \frac{27}{8}$

**Exercise 5.16**
$\frac{24}{75} \div \frac{8}{15}$

**Exercise 5.17** ***(Solution on p. 31.)***
$\frac{57}{8} \div \frac{7}{8}$

**Exercise 5.18**
$\frac{7}{10} \div \frac{10}{7}$

**Exercise 5.19** ***(Solution on p. 31.)***
$\frac{3}{8} + \frac{2}{8}$

**Exercise 5.20**
$\frac{3}{11} + \frac{4}{11}$

**Exercise 5.21** ***(Solution on p. 31.)***
$\frac{5}{12} + \frac{7}{12}$

**Exercise 5.22**
$\frac{11}{16} - \frac{2}{16}$

**Exercise 5.23** ***(Solution on p. 31.)***
$\frac{15}{23} - \frac{2}{23}$

**Exercise 5.24**
$\frac{3}{11} + \frac{1}{11} + \frac{5}{11}$

**Exercise 5.25** ***(Solution on p. 31.)***

$\frac{16}{20}+\frac{1}{20}+\frac{2}{20}$

**Exercise 5.26**

$\frac{3}{8}+\frac{2}{8}-\frac{1}{8}$

**Exercise 5.27** ***(Solution on p. 31.)***

$\frac{11}{16}+\frac{9}{16}-\frac{5}{16}$

**Exercise 5.28**

$\frac{1}{2}+\frac{1}{6}$

**Exercise 5.29** ***(Solution on p. 31.)***

$\frac{1}{8}+\frac{1}{2}$

**Exercise 5.30**

$\frac{3}{4}+\frac{1}{3}$

**Exercise 5.31** ***(Solution on p. 31.)***

$\frac{5}{8}+\frac{2}{3}$

**Exercise 5.32**

$\frac{6}{7}-\frac{1}{4}$

**Exercise 5.33** ***(Solution on p. 31.)***

$\frac{8}{15}-\frac{3}{10}$

**Exercise 5.34**

$\frac{1}{15}+\frac{5}{12}$

**Exercise 5.35** ***(Solution on p. 31.)***

$\frac{25}{36}-\frac{7}{10}$

**Exercise 5.36**

$\frac{9}{28}-\frac{4}{45}$

**Exercise 5.37** ***(Solution on p. 31.)***

$\frac{8}{15}-\frac{3}{10}$

**Exercise 5.38**

$\frac{1}{16}+\frac{3}{4}-\frac{3}{8}$

**Exercise 5.39** ***(Solution on p. 31.)***

$\frac{8}{3}-\frac{1}{4}+\frac{7}{36}$

**Exercise 5.40**

$\frac{3}{4}-\frac{3}{22}+\frac{5}{24}$

# Solutions to Exercises in Chapter 5

**Solution to Exercise 5.1 (p. 28)**
$\frac{4}{9}$

**Solution to Exercise 5.3 (p. 29)**
$\frac{1}{3}$

**Solution to Exercise 5.5 (p. 29)**
$\frac{5}{12}$

**Solution to Exercise 5.7 (p. 29)**
$\frac{9}{10}$

**Solution to Exercise 5.9 (p. 29)**
$1$

**Solution to Exercise 5.11 (p. 29)**
$\frac{2}{3}$

**Solution to Exercise 5.13 (p. 29)**
$\frac{10}{9}$

**Solution to Exercise 5.15 (p. 29)**
$\frac{10}{9}$

**Solution to Exercise 5.17 (p. 29)**
$\frac{57}{7}$

**Solution to Exercise 5.19 (p. 29)**
$\frac{5}{8}$

**Solution to Exercise 5.21 (p. 29)**
$1$

**Solution to Exercise 5.23 (p. 29)**
$\frac{13}{23}$

**Solution to Exercise 5.25 (p. 30)**
$\frac{19}{20}$

**Solution to Exercise 5.27 (p. 30)**
$\frac{15}{16}$

**Solution to Exercise 5.29 (p. 30)**
$\frac{5}{8}$

**Solution to Exercise 5.31 (p. 30)**
$\frac{31}{24}$

**Solution to Exercise 5.33 (p. 30)**
$\frac{5}{6}$

**Solution to Exercise 5.35 (p. 30)**
$\frac{-1}{180}$

**Solution to Exercise 5.37 (p. 30)**
$\frac{7}{30}$

**Solution to Exercise 5.39 (p. 30)**
$\frac{47}{18}$

# Chapter 6

# Adding and Subtracting Fractions with Like and Unlike Denominators, and LCD[1]

## 6.1 Adding Fractions with Like Denominators

To add two or more fractions that have the same denominators, add the numerators and place the resulting sum over the **common denominator**. Reduce, if necessary.

**Example 6.1**

Find the following sums.

$\frac{3}{7} + \frac{2}{7}$

The denominators are the same.

Add the numerators and place the sum over the common denominator, 7.

$\frac{3}{7} + \frac{2}{7} = \frac{3+2}{7} = \frac{5}{7}$

When necessary, reduce the result.

**Example 6.2**

$\frac{1}{8} + \frac{3}{8} = \frac{1+3}{8} = \frac{4}{8} = \frac{1}{2}$

NOTE: We do not add denominators.

**Example 6.3**

To see what happens if we **mistakenly** add the denominators as well as the numerators, let's add $\frac{1}{2}$ and $\frac{1}{2}$.

Adding the numerators and **mistakenly** adding the denominators produces:

$\frac{1}{2} + \frac{1}{2} = \frac{1+1}{2+2} = \frac{2}{4} = \frac{1}{2}$

This means that $\frac{1}{2} + \frac{1}{2}$is the same as $\frac{1}{2}$ , which is preposterous! **We do not add denominators**.

### 6.1.1 Adding Fractions with Like Denominators - Exercises

#### 6.1.1.1 Find the following sums.

**Exercise 6.1** ***(Solution on p. 38.)***

$\frac{3}{8} + \frac{3}{8}$

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[1] This content is available online at <http://cnx.org/content/m26339/1.1/>.

**Exercise 6.2** *(Solution on p. 38.)*

$\frac{7}{11} + \frac{4}{11}$

**Exercise 6.3** *(Solution on p. 38.)*

$\frac{15}{20} + \frac{1}{20} + \frac{2}{20}$

## 6.2 Subtracting Fractions with Like Denominators

To subtract two or more fractions that have the same denominators, subtract the numerators and place the resulting difference over the **common denominator**. Reduce, if necessary.

**Example 6.4**

Find the following differences.

$\frac{3}{5}$ - $\frac{1}{5}$

The denominators are the same.

Subtract the numerators and place the difference over the common denominator, 5.

$\frac{3}{5}$ - $\frac{1}{5} = \frac{3-1}{5} = \frac{2}{5}$

When necessary, reduce the result.

**Example 6.5**

$\frac{8}{6}$ - $\frac{2}{6} = \frac{6}{6} = 1$

NOTE: We do not subtract denominators.

**Example 6.6**

To see what happens if we **mistakenly** subtract the denominators as well as the numerators, let's subtract

$\frac{7}{15}$ - $\frac{4}{15}$.

Subtracting the numerators and **mistakenly** subtracting the denominators produces:

$\frac{7}{15}$ - $\frac{4}{15} = \frac{7-4}{15-15} = \frac{3}{0}$

We end up dividing by zero, which is undefined. **We do not subtract denominators.**

### 6.2.1 Subtracting Fractions with Like Denominators - Exercises

#### 6.2.1.1 Find the following differences.

**Exercise 6.4** *(Solution on p. 38.)*

$\frac{5}{12}$ - $\frac{1}{12}$

**Exercise 6.5** *(Solution on p. 38.)*

$\frac{3}{16}$ - $\frac{3}{16}$

**Exercise 6.6** *(Solution on p. 38.)*

$\frac{16}{5}$ - $\frac{1}{5}$ - $\frac{2}{5}$

## 6.3 Adding and Subtracting Fractions with Unlike Denominators

**Basic Rule: Fractions can only be added or subtracted conveniently if they have like denominators.**

To see why this rule makes sense, let's consider the problem of adding a quarter and a dime.

A quarter is $\frac{1}{4}$ of a dollar.

A dime is $\frac{1}{10}$ of a dollar.

We know that 1 quarter + 1 dime = 35 cents. How do we get to this answer by adding $\frac{1}{4}$ and $\frac{1}{10}$ ?

We convert them to quantities of the same denomination.

A quarter is equivalent to 25 cents, or $\frac{25}{100}$.

A dime is equivalent to 10 cents, or $\frac{10}{100}$.

By converting them to quantities of the same denomination, we can add them easily:

$\frac{25}{100} + \frac{10}{100} = \frac{35}{100}$.

**Same denomination → same denominator**

If the denominators are not the same, make them the same by building up the fractions so that they both have a common denominator. A common denominator can always be found by multiplying all the denominators, but it is not necessarily the Least Common Denominator.

## 6.4 Least Common Denominator (LCD)

**The LCD is the smallest number that is evenly divisible by all the denominators.**

It is the **least common multiple** of the denominators.

The LCD is the product of all the **prime factors** of all the denominators, each factor taken the greatest number of times that it appears in any single denominator.

### 6.4.1 Finding the LCD

**Example 6.7**

Find the sum of these unlike fractions.

$\frac{1}{12} + \frac{4}{15}$

Factor the denominators:

$12 = 2 \times 2 \times 3$

$15 = 3 \times 5$

What is the greatest number of times the prime factor 2 appear in any single denominator? **Answer: 2times.** That is the number of times the prime factor 2 will appear as a factor in the LCD.

What is the greatest number of times the prime factor 3 appear in any single denominator? **Answer: 1 time.** That is the number of times the prime factor 3 will appear as a factor in the LCD.

What is the greatest number of times the prime factor 5 appear in any single denominator? **Answer: 1 time.** That is the number of times the prime factor 5 will appear as a factor in the LCD.

So we assemble the LCD by multiplying each prime factor by the number of times it appears in a single denominator, or:

$2 \times 2 \times 3 \times 5 = 60$

**60 is the Least Common Denominator (the Least Common Multiple of 12 and 15)**.

### 6.4.2 Building up the Fractions

To create fractions with like denominators, we now multiply the numerators by whatever factors are missing when we compare the original denominator to the new LCD.

**Example 6.8**

In the fraction $\frac{1}{12}$, we multiply the denominator 12 by 5 to get the LCD of 60. Therefore we multiply the numerator 1 by the same factor (5).

$\frac{1}{12} \times \frac{5}{5} = \frac{5}{60}$

Similarly,

$\frac{4}{15} \times \frac{4}{4} = \frac{16}{60}$

### 6.4.3 Adding the Built Up Fractions

**Example 6.9**

We can now add the two fractions because they have like denominators:

$\frac{5}{60} + \frac{16}{60} = \frac{21}{60}$

Reduce the result: $\frac{21}{60} = \frac{7}{20}$

### 6.4.4 Adding and Subtracting Fractions with Unlike Denominators - Exercises

#### 6.4.4.1 Find the following sums and differences.

**Exercise 6.7** ***(Solution on p. 38.)***

$\frac{1}{6} + \frac{3}{4}$

**Exercise 6.8** ***(Solution on p. 38.)***

$\frac{5}{9}$ - $\frac{5}{12}$

**Exercise 6.9** ***(Solution on p. 38.)***

$\frac{15}{16} + \frac{1}{2}$ - $\frac{3}{4}$

## 6.5 Module Review Exercises

**Exercise 6.10** ***(Solution on p. 38.)***

$\frac{9}{15} + \frac{4}{15}$

**Exercise 6.11** ***(Solution on p. 38.)***

$\frac{7}{10}$ - $\frac{3}{10} + \frac{11}{10}$

**Exercise 6.12** ***(Solution on p. 38.)***

Find the total length of the screw in this diagram:

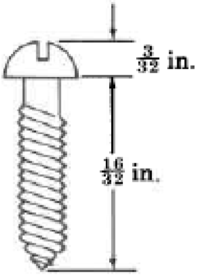


**Figure 6.1**

**Exercise 6.13** *(Solution on p. 38.)*

$\frac{5}{2} + \frac{16}{2} - \frac{3}{2}$

**Exercise 6.14** *(Solution on p. 38.)*

$\frac{3}{4} + \frac{1}{3}$

**Exercise 6.15** *(Solution on p. 38.)*

Two months ago, a woman paid off $\frac{3}{24}$ of a loan. One month ago, she paid off $\frac{4}{24}$ of the loan. This month she will pay off $\frac{5}{24}$ of the total loan. At the end of this month, how much of her total loan will she have paid off?

**Exercise 6.16** *(Solution on p. 38.)*

$\frac{8}{3} - \frac{1}{4} + \frac{7}{36}$

## Solutions to Exercises in Chapter 6

**Solution to Exercise 6.1 (p. 33)**
$\frac{6}{8} = \frac{3}{4}$
**Solution to Exercise 6.2 (p. 34)**
$\frac{11}{11} = 1$
**Solution to Exercise 6.3 (p. 34)**
$\frac{18}{20} = \frac{9}{10}$
**Solution to Exercise 6.4 (p. 34)**
$\frac{4}{12} = \frac{1}{3}$
**Solution to Exercise 6.5 (p. 34)**
Result is 0
**Solution to Exercise 6.6 (p. 34)**
Result is $\frac{13}{5}$
**Solution to Exercise 6.7 (p. 36)**
Result is $\frac{11}{12}$
**Solution to Exercise 6.8 (p. 36)**
Result is $\frac{5}{36}$
**Solution to Exercise 6.9 (p. 36)**
Result is $\frac{35}{16}$
**Solution to Exercise 6.10 (p. 36)**
Result is $\frac{13}{15}$
**Solution to Exercise 6.11 (p. 36)**
Result is $\frac{15}{10}$ (reduce to $1\ \frac{1}{2}$ )
**Solution to Exercise 6.12 (p. 36)**
Total length is $\frac{19}{32}$ in.
**Solution to Exercise 6.13 (p. 37)**
Result is $\frac{18}{2}$ (reduce to 9)
**Solution to Exercise 6.14 (p. 37)**
Result is $\frac{13}{12}$
**Solution to Exercise 6.15 (p. 37)**
She will have paid off $\frac{12}{24}$, or $\frac{1}{2}$ of the total loan.
**Solution to Exercise 6.16 (p. 37)**
Result is $\frac{94}{36}$ (reduce to $\frac{47}{18}$ )

# Chapter 7

# Signed Numbers: Absolute Value[1]

## 7.1 Section Overview

- Geometric Definition of Absolute Value
- Algebraic Definition of Absolute Value

## 7.2 Geometric Definition of Absolute Value

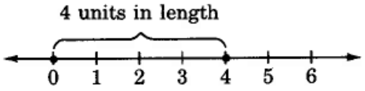
**Absolute Value-Geometric Approach**
Geometric definition of absolute value:
The **absolute value** of a number $a$, denoted $\mid a \mid$, is the distance from a to 0 on the number line.

Absolute value answers the question of "how far," and not "which way." The phrase "how far" implies "length" and *length is always a nonnegative quantity*. Thus, the absolute value of a number is a nonnegative number.

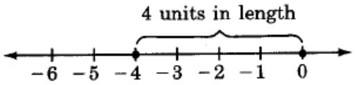
### 7.2.1 Sample Set A

Determine each value.

**Example 7.1**
$\mid 4 \mid = 4$



**Example 7.2**
$\mid -4 \mid = 4$



**Example 7.3**
$\mid 0 \mid = 0$

**Example 7.4**
$- \mid 5 \mid = -5$. The quantity on the left side of the equal sign is read as "negative the absolute value of 5." The absolute value of 5 is 5. Hence, negative the absolute value of 5 is -5.

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[1] This content is available online at <http://cnx.org/content/m35030/1.3/>.

**Example 7.5**
$- \mid -3 \mid = -3$. The quantity on the left side of the equal sign is read as "negative the absolute value of -3." The absolute value of -3 is 3. Hence, negative the absolute value of -3 is $-(3) = -3$.

### 7.2.2 Practice Set A

By reasoning geometrically, determine each absolute value.

**Exercise 7.1** ***(Solution on p. 44.)***
$\mid 7 \mid$

**Exercise 7.2** ***(Solution on p. 44.)***
$\mid -3 \mid$

**Exercise 7.3** ***(Solution on p. 44.)***
$\mid 12 \mid$

**Exercise 7.4** ***(Solution on p. 44.)***
$\mid 0 \mid$

**Exercise 7.5** ***(Solution on p. 44.)***
$- \mid 9 \mid$

**Exercise 7.6** ***(Solution on p. 44.)***
$- \mid -6 \mid$

## 7.3 Algebraic Definition of Absolute Value

From the problems in Section 7.2.1 (Sample Set A), we can suggest the following algebraic definition of absolute value. Note that the definition has two parts.

**Absolute Value—Algebraic Approach**
Algebraic definition of absolute value
The absolute value of a number $a$ is

$$|a| = \{ \begin{array}{cc} a, & \text{if } a \geq 0 \\ -a, & \text{if } < 0 \end{array}$$

The algebraic definition takes into account the fact that the number $a$ could be either positive or zero ($a \geq 0$) or negative ($a < 0$).

1. If the number $a$ is positive or zero ($a \geq 0$), the upper part of the definition applies. The upper part of the definition tells us that if the number enclosed in the absolute value bars is a nonnegative number, the absolute value of the number is the number itself.
2. The lower part of the definition tells us that if the number enclosed within the absolute value bars is a negative number, the absolute value of the number is the opposite of the number. The opposite of a negative number is a positive number.

NOTE: The definition says that the vertical absolute value lines may be eliminated only if we know whether the number inside is positive or negative.

### 7.3.1 Sample Set B

Use the algebraic definition of absolute value to find the following values.

**Example 7.6**
$|\ 8\ |$. The number enclosed within the absolute value bars is a nonnegative number, so the upper part of the definition applies. This part says that the absolute value of 8 is 8 itself.

$|\ 8\ |= 8$

**Example 7.7**
$|\ -3\ |$. The number enclosed within absolute value bars is a negative number, so the lower part of the definition applies. This part says that the absolute value of -3 is the opposite of -3, which is $-(-3)$. By the definition of absolute value and the double-negative property,

$|\ -3\ |= -(-3) = 3$

### 7.3.2 Practice Set B

Use the algebraic definition of absolute value to find the following values.

**Exercise 7.7** ***(Solution on p. 44.)***
$|\ 7\ |$

**Exercise 7.8** ***(Solution on p. 44.)***
$|\ 9\ |$

**Exercise 7.9** ***(Solution on p. 44.)***
$|\ -12\ |$

**Exercise 7.10** ***(Solution on p. 44.)***
$|\ -5\ |$

**Exercise 7.11** ***(Solution on p. 44.)***
$-\ |\ 8\ |$

**Exercise 7.12** ***(Solution on p. 44.)***
$-\ |\ 1\ |$

**Exercise 7.13** ***(Solution on p. 44.)***
$-\ |\ -52\ |$

**Exercise 7.14** ***(Solution on p. 44.)***
$-\ |\ -31\ |$

## 7.4 Exercises

Determine each of the values.

**Exercise 7.15** ***(Solution on p. 44.)***
$|\ 5\ |$

**Exercise 7.16**
$|\ 3\ |$

**Exercise 7.17** ***(Solution on p. 44.)***
$|\ 6\ |$

**Exercise 7.18**
$|\ -9\ |$

**Exercise 7.19** ***(Solution on p. 44.)***
$|\ -1\ |$

**Exercise 7.20**
$|-4|$

**Exercise 7.21** *(Solution on p. 44.)*
$-|3|$

**Exercise 7.22**
$-|7|$

**Exercise 7.23** *(Solution on p. 44.)*
$-|-14|$

**Exercise 7.24**
$|0|$

**Exercise 7.25** *(Solution on p. 44.)*
$|-26|$

**Exercise 7.26**
$-|-26|$

**Exercise 7.27** *(Solution on p. 44.)*
$-(-|4|)$

**Exercise 7.28**
$-(-|2|)$

**Exercise 7.29** *(Solution on p. 44.)*
$-(-|-6|)$

**Exercise 7.30**
$-(-|-42|)$

**Exercise 7.31** *(Solution on p. 44.)*
$|5|-|-2|$

**Exercise 7.32**
$|-2|^3$

**Exercise 7.33** *(Solution on p. 44.)*
$|-(2\cdot 3)|$

**Exercise 7.34**
$|-2|-|-9|$

**Exercise 7.35** *(Solution on p. 44.)*
$(|-6|+|4|)^2$

**Exercise 7.36**
$(|-1|-|1|)^3$

**Exercise 7.37** *(Solution on p. 44.)*
$(|4|+|-6|)^2-(|-2|)^3$

**Exercise 7.38**
$-[|-10|-6]^2$

**Exercise 7.39** *(Solution on p. 45.)*
$-\{-[-|-4|+|-3|]^3\}^2$

**Exercise 7.40**
A Mission Control Officer at Cape Canaveral makes the statement “lift-off, $T$ minus 50 seconds.” How long is it before lift-off?

**Exercise 7.41** *(Solution on p. 45.)*
Due to a slowdown in the industry, a Silicon Valley computer company finds itself in debt \$2,400,000. Use absolute value notation to describe this company’s debt.

**Exercise 7.42**

A particular machine is set correctly if upon action its meter reads 0. One particular machine has a meter reading of $-1.6$ upon action. How far is this machine off its correct setting?

### 7.4.1 Exercises for Review

**Exercise 7.43** *(Solution on p. 45.)*

( here[2]) Find the sum: $\frac{9}{70} + \frac{5}{21} + \frac{8}{15}$.

**Exercise 7.44**

( here[3]) Find the value of $\frac{\frac{3}{10} + \frac{4}{12}}{\frac{19}{20}}$.

**Exercise 7.45** *(Solution on p. 45.)*

( here[4]) Convert $3.2\frac{3}{5}$ to a fraction.

**Exercise 7.46**

( here[5]) The ratio of acid to water in a solution is $\frac{3}{8}$. How many mL of acid are there in a solution that contain 112 mL of water?

**Exercise 7.47** *(Solution on p. 45.)*

( here[6]) Find the value of $-6 - (-8)$.

---

[2] "Addition and Subtraction of Fractions, Comparing Fractions, and Complex Fractions: Addition and Subtraction of Fractions with Unlike Denominators" <http://cnx.org/content/m34935/latest/>

[3] "Addition and Subtraction of Fractions, Comparing Fractions, and Complex Fractions: Complex Fractions" <http://cnx.org/content/m34941/latest/>

[4] "Decimals: Converting a Decimal to a Fraction" <http://cnx.org/content/m34958/latest/>

[5] "Ratios and Rates: Proportions" <http://cnx.org/content/m34981/latest/>

[6] "Signed Numbers: Signed Numbers" <http://cnx.org/content/m35029/latest/>

## Solutions to Exercises in Chapter 7

**Solution to Exercise 7.1 (p. 40)**
7

**Solution to Exercise 7.2 (p. 40)**
3

**Solution to Exercise 7.3 (p. 40)**
12

**Solution to Exercise 7.4 (p. 40)**
0

**Solution to Exercise 7.5 (p. 40)**
-9

**Solution to Exercise 7.6 (p. 40)**
-6

**Solution to Exercise 7.7 (p. 41)**
7

**Solution to Exercise 7.8 (p. 41)**
9

**Solution to Exercise 7.9 (p. 41)**
12

**Solution to Exercise 7.10 (p. 41)**
5

**Solution to Exercise 7.11 (p. 41)**
-8

**Solution to Exercise 7.12 (p. 41)**
-1

**Solution to Exercise 7.13 (p. 41)**
-52

**Solution to Exercise 7.14 (p. 41)**
-31

**Solution to Exercise 7.15 (p. 41)**
5

**Solution to Exercise 7.17 (p. 41)**
6

**Solution to Exercise 7.19 (p. 41)**
1

**Solution to Exercise 7.21 (p. 42)**
-3

**Solution to Exercise 7.23 (p. 42)**
-14

**Solution to Exercise 7.25 (p. 42)**
26

**Solution to Exercise 7.27 (p. 42)**
4

**Solution to Exercise 7.29 (p. 42)**
6

**Solution to Exercise 7.31 (p. 42)**
3

**Solution to Exercise 7.33 (p. 42)**
6

**Solution to Exercise 7.35 (p. 42)**
100

**Solution to Exercise 7.37 (p. 42)**
92

**Solution to Exercise 7.39 (p. 42)**
-1

**Solution to Exercise 7.41 (p. 42)**
$-\$ \mid -2,400,000 \mid$

**Solution to Exercise 7.43 (p. 43)**
$\frac{9}{10}$

**Solution to Exercise 7.45 (p. 43)**
$3\frac{13}{50}$ or $\frac{163}{50}$

**Solution to Exercise 7.47 (p. 43)**
2

# Index of Keywords and Terms

**Keywords** are listed by the section with that keyword (page numbers are in parentheses). Keywords do not necessarily appear in the text of the page. They are merely associated with that section. *Ex.* apples, § 1.1 (1) **Terms** are referenced by the page they appear on. *Ex.* apples, 1

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algebra, § 1(1), § 2(7), § 3(13), § 4(19), § 5(25)

**B** Burzynski, § 7(39)

**C** common denominator, 33, 34
composite number, § 2(7)
composite number., 7

**E** elementary, § 1(1), § 2(7), § 3(13), § 4(19), § 5(25)
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**U** unlike denominators, § 6(33)

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